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5.8 FLOW ROUTING

5.8.1 Introduction

Flow routing is a mathematical method (model) to predict the changing magnitude, speed, and shape of a flood wave at one or more locations along waterways such as rivers, reservoirs, canals, or estuaries. The flood wave can emanate from precipitation runoff (rainfall or snowmelt), reservoir releases (spillway flows or dam-failures), and tides (astronomical and/or wind-generated).

Flow routing has long been of vital concern and many ways have been developed to predict the characteristic features of a flood wave in order to improve the transport of water through natural or man-made waterways and to determine necessary actions to protect life and property from the effects of flooding. Commencing with investigations by Newton (1687), Laplace (1776), Poisson (1816), Boussinesq (1871), and culminating in the one-dimensional equations of unsteady flow derived by Barré de Saint-Venant (1871), the theoretical foundation for flow routing was essentially achieved. The original Saint-Venant equations are the conservation of mass equation:

$$\partial(AV)/\partial x + \partial A/\partial t = 0 \tag{1}$$

and the conservation of momentum equation:

$$\partial V/\partial t + V\,\partial V/\partial x + g\left(\partial h/\partial x + S_f\right) = 0 \tag{2}$$

in which t is time, x is distance along the longitudinal axis of the waterway, A is cross-sectional area, V is velocity, g is the gravity acceleration constant, and h is the water surface elevation above a datum. S_f is the friction slope which may now be evaluated using a steady flow empirical formula such as the Manning equation (Manning, 1889; Chow, 1959), i.e.,

$$Q = \mu/n\ A R^{2/3} S_o^{1/2} \tag{3}$$

in which Q=AV is discharge or flow, $R=A/P$ is the hydraulic radius and P is the wetted perimeter of the cross section, S_o is the channel bottom slope (dimensionless), μ is a units conversion factor, i.e., 1.49 for U.S. units or 1.0 for SI, and n is the Manning roughness (friction) coefficient. Equations (1) and (2) are quasi-linear hyperbolic partial differential equations with two dependent parameters (V and h) and two independent parameters (x and t). A is a known function of h, and S_f is a known function of V and h. Derivations of the Saint-Venant equations can be found in the following references: Stoker (1957), Henderson (1966), Strelkoff (1969), and Liggett (1975).

Due to the mathematical complexity of the Saint-Venant equations (no analytical solution is known), simplifications were necessary to obtain feasible solutions for the salient characteristics of a propagating flood wave. This approach produced a profusion of simplified flow routing models. The simplified flow routing models may be categorized as: (I) purely empirical; (II) storage routing, based on the conservation of mass and an approximate relation between flow and storage; and (III) hydraulic, i.e., based on the conservation of mass and a simplified form of the conservation of momentum equation (2).

Categories (I) and (II) are further classified as lumped flow routing techniques in which the flow is computed as a function of time, only at the most downstream location of routing reaches along the waterway. Category (III) can be classified as distributed flow routing techniques in which flow and depth or water-surface elevation are computed as a function of time at frequent locations within routing reaches along the waterway. During the last two decades dynamic hydraulic distributed flow routing methods based on numerical solutions of the complete Saint-Venant equations have become economically feasible as a result of advances in computing equipment and improved numerical solution techniques. Following is a brief description of some of the more popular storage routing models as well as both simplified and dynamic hydraulic flow routing models.

5.8.2 Storage Routing Models

Significant river improvement projects in the early 1900s provided the impetus for development of an array of simplified flow routing methods. These have been termed storage routing models. They are based on the conservation of mass equation (1) written in the following form:

$$\bar{I} - \bar{O} = \Delta \bar{S} / \Delta t \tag{4}$$

in which ΔS is the change in storage within the routing reach during a Δt time increment, $\bar{I} = 0.5\,[I(t) + I(t+\Delta t)]$, and $\bar{O} = 0.5\,[O(t) + O(t+\Delta t)]$; the storage $(\bar{S})$ is assumed to be related to inflow (I) and/or outflow (O), i.e.,

$$\bar{S} = \bar{K}\,[\bar{X}\,\bar{I} + (1 - \bar{X})\,\bar{O}] \tag{5}$$

in which $\bar{K}$ is a storage constant with dimensions of time, and $\bar{X}$ is a weighting coefficient, $0 \le \bar{X} \le 1$. Storage routing models are limited to typical flood routing applications where the outflow and water-surface elevation relation is essentially single-valued, and the waterways are not mild sloping ($S_o > 0.002$). Thus backwater effects from tides, significant tributary inflow, and dams or bridges are not considered by these models, nor are they well-suited for rapidly changing unsteady flows such as dam-break flood waves, reservoir power releases, or hurricane storm surges. Generally, storage routing models have two parameters which can be calibrated to effectively reproduce the flood wave speed and its attenuated peak. The calibration requires that most storage routing model applications be limited to where observed inflow-outflow hydrographs exist. When using the observed hydographs to calibrate the routing coefficients, variations in flood wave shapes within the observed data set are not considered, and only the average wave shape is reflected in the fitted routing coefficients.

Reservoir Storage Routing Model

Storage routing models applicable to reservoirs, which have essentially level water-surface profiles, can be developed by assuming $\bar{X}$ to be zero in equation (5), i.e. storage is dependent only on outflow. Expressing the term $\Delta\bar{S}/\Delta t$ in equation (4) as the product of reservoir surface area (S_a) which is a known function of water-surface elevation (h) and the change of h over a j Δt time step, i.e.

$$\Delta\bar{S}/\Delta t \;=\; 0.5\,(S_a^j + S_a^{j+1})(h^{j+1} - h^j)/\Delta t^j \tag{6}$$

Now denoting $\bar{O}$ (outflow) as $\bar{Q}$ (discharge), the following reservoir routing model (Fread, 1977) is obtained:

$$0.5\,(I^j + I^{j+1}) \;-\; 0.5\,(Q^j + Q^{j+1}) \;-\; 0.5\,(S_a^j + S_a^{j+1})(h^{j+1} - h^j)/\Delta t^j \;=\; 0 \tag{7}$$

The inflow (I) at times j and $j+1$ are known from the specified inflow hydrograph, the outflow (Q^j) at time j can be computed from the known water-surface elevation (h^j) and an appropriate spillway discharge equation. The surface area (S_a^j) can be determined from the known value of h^j. The unknowns in the equation consist of h^{j+1}, Q^{j+1}, S_a^{j+1}; the latter two are known nonlinear functions of h^{j+1}. Hence, equation (7) can be solved for h^{j+1} by an iterative method such as Newton-Raphson, i.e.,

$$h_{k+1}^{j+1} \;=\; h_k^{j+1} \;-\; f(h_k^{j+1})/f'(h_k^{j+1}) \tag{8}$$

in which k is the iteration counter; and $f(h_k^{j+1})$ is the left-hand side of equation (7) evaluated with the first estimate for h_k^{j+1}, which for $k=1$ is either h^j or a linear extrapolated estimate of h^{j+1}; $f'(h_k^{j+1})$ is the derivative of equation (7) with respect to h^{j+1}. It can be approximated by using a numerical derivative as follows:

$$f'(h_k^{j+1}) = \left[f(h_k^{j+1} + \epsilon) - f(h_k^{j+1} - \epsilon)\right] / \left[(h_k^{j+1} + \epsilon) - (h_k^{j+1} - \epsilon)\right] \tag{9}$$

in which ϵ is a small value, say 0.1 ft (0.03m). Using equation (8), only one or two iterations are usually required to solve equation (7) for h^{j+1}. Initially, the reservoir pool elevation (h^j) must be known to start the computational process. Once h^{j+1} is obtained Q^{j+1} can be computed from the spillway discharge equation, $Q = f(h_k^{j+1})$.

Level-pool routing is less accurate as the reservoir length increases, as the reservoir mean depth decreases, and as the time of rise of the inflow hydrograph decreases (Fread, 1992). This inaccuracy can have significant economic effects on water control management practices (Sayed and Howard, 1983).

Muskingum Model

A widely used hydrologic flow routing model is the Muskingum model developed by using equation (5), with nonzero values for both $\bar{K}$ and $\bar{X}$, for the storage relationship. Substituting this information into equation (4), the following is obtained for computing $O(t+\Delta t)$:

$$O(t+\Delta t) = C_1 I(t+\Delta t) + C_2 I(t) + C_3 O(t) + C_4 \tag{10}$$

where:

$$C_o = \bar{K} - \bar{K}\bar{X} + \Delta t/2 \tag{11}$$

$$C_1 = -(\bar{K}\bar{X} - \Delta t/2)/C_0 \tag{12}$$

$$C_2 = (\bar{K}\bar{X} + \Delta t/2)/C_0 \tag{13}$$

$$C_3 = (\bar{K} - \bar{K}\bar{X} - \Delta t/2)/C_0 \tag{14}$$

$$C_4 = 0.5\left[q(t) + q(t+\Delta t)\right]\Delta x\, \Delta t/C_0 \tag{15}$$

and where $C_1 + C_2 + C_3 = 1$ and $\bar{K}/3 \le \Delta t \le \bar{K}$ is usually the range for Δt.

Equation (10), which has been expanded to include the effects of lateral inflow (q) along the Δx routing reach, is the widely used Muskingum routing model first developed by McCarthy (1938). The parameters $\bar{K}$ and $\bar{X}$ are determined from observed inflow-outflow hydrographs using least-squares or its equivalent, the graphical method or other techniques (Singh and McCann, 1980). Among the many descriptions and variations of the Muskingum model are: Chow (1964); Chow et al. (1988); Strupczewski and Kundzewicz (1980); Dooge et al. (1982); and Linsley et al. (1986).

Muskingum-Cunge Model

A significant improvement of the Muskingum model was developed by Cunge (1969) known as the Muskingum-Cunge model. This increased the Muskingum model's accuracy, and made it applicable in situations where observed inflow and outflow hydrographs were not available for calibration, and enabled it to be changed from a lumped to a distributed flow routing model. Cunge derived equation (10) using the assumption of a single-valued Q (h) relation, the classical kinematic wave equation (see equation (25)), and applying a four-point implicit finite-difference approximation technique. Equation (10) is rewritten where the flows$I(t)$, $I(t+\Delta t)$, $O(t)$ and $O(t+\Delta t)$ are replaced by Q_i^j, Q_i^{j+1}, Q_{i+1}^j, and Q_{i+1}^{j+1}, respectively, i.e.,

$$Q_{i+1}^{j+1} = C_1 Q_i^{j+1} + C_2 Q_i^j + C3\, Q_{i+1}^j + C_4 \qquad (16)$$

but the following expressions for $\bar{K}$ and $\bar{X}$ are determined:

$$\bar{K} = \Delta x / c \qquad (17)$$

$$\bar{X} = 0.5\left[1 - \bar{Q}/(\bar{c}\bar{B}S_e\ \Delta x)\right] \qquad (18)$$

where:

$$\bar{c} = dQ/dA \qquad (19)$$

in which $\bar{c}$ is the kinematic wave speed, Δx is the reach length, and S_e is the energy slope approximated by evaluating S_o in equation (3) for the initial flow condition. The bar (-) indicates the variable is averaged over the Δx reach and over the Δt time step. Equation (19) may be expressed in an alternative form, i.e.,

$$\bar{c} = K'\bar{Q}/\bar{A} \qquad (20)$$

where:

$$K' = 5/3 - 2/3\,(d\bar{B}/dy)\,\bar{A}/(\bar{B})^2 \qquad (21)$$

in which A is the cross-sectional area; B is the channel width at the water surface, h is the water-surface elevation of the flow, and the Manning equation is used to relate discharge (Q) and depth or water-surface elevation (h). Depending on the cross-section shape, K' may have values in the range, $4/3 \leq K' \leq 5/3$; the upper value is associated with either a very wide or rectangular channel. Selection of the appropriate time step Δt is given by:

$$\Delta t \leq T_r / M \tag{22}$$

where T_r is the time of rise in hours of the inflow hydrograph and M is an integer ($10 \leq M \leq 20$) whose value depends on the extent of variation in the inflow hydrograph. The selection of Δx affects the accuracy of the solution. It is related to Δt and is limited by the following inequality (Jones, 1981):

$$\Delta x \leq 0.5\, c\, \Delta t \left[1 + \left(1 + 1.5\, Q/(B\, c^2 S_o\, \Delta t)\right)^{1/2}\right] \tag{23}$$

While the Muskingum-Cunge model does not require observed inflow-outflow hydrographs to establish the routing coefficients as required in the Muskingum model, best results are obtained if the wave speed (c) is determined from actual flow data. Also, the model is restricted to applications where backwater is not significant and discharge-water elevation rating curves do not have significant loops and discharge hydrographs are not rapidly changing with time such as dam-break floods. Nonetheless, the Muskingum-Cunge model (Miller and Cunge, 1975; Ponce and Yevjevich, 1978) is a highly versatile simplified routing model.

5.8.3 Simplified Hydraulic Routing Models

Prior to computers, or more recently the feasible economical availability of such computational resources, the inability to obtain feasible numerical solutions to the complete Saint-Venant equations resulted in the development of several simplified distributed hydraulic routing models. They are based on the mass conservation equation (1) and various simplifications of the momentum equation (2).

<u>Kinematic Wave Model</u>

The most simple type of distributed hydraulic routing model is the kinematic wave model. It is based on the following simplified form of the momentum equation (2):

$$S_f - S_o = 0 \tag{24}$$

in which S_o is the bottom slope of the channel (waterway) and a component of the term, $\partial h/\partial x = \partial y/\partial x - S_o$, in which $\partial y/\partial x$ is assumed to be zero. This assumes that the momentum of the unsteady flow is the same as that of steady, uniform flow described by the Manning equation or a similar expression in which discharge is a single-valued function of depth, i.e., $\partial Q/\partial A = dQ/dA = c$. Also, since $\partial A/\partial t = \partial A/\partial Q \cdot \partial Q/\partial t$ and $Q = AV$, equation (1) can be expanded into the classical kinematic wave equation, i.e.,

$$\partial Q/\partial t + c\, \partial Q/\partial x = 0 \tag{25}$$

in which the kinematic wave velocity or celerity (c) is defined by equation (20).

Solutions for the kinematic wave equation (25) can be achieved using the method of characteristics or directly by finite-difference approximation techniques of either explicit or implicit types (Chow et al., 1988; Hydrologic Engr. Ctr., 1981; Linsley et al., 1986). The kinematic wave equation does not theoretically account for hydrograph (wave) attenuation. It is only through the numerical error associated with the finite-difference solution that attenuation of the hydrograph peak is achieved. Kinematic wave models are limited to applications where single-value, stage-discharge ratings exist--where there are no loop-ratings--and where backwater effects are insignificant. Since, in kinematic wave models, flow disturbances can propagate only in the downstream direction, reverse (negative) flows cannot be predicted. Kinematic wave models are appropriately used as components of hydrologic watershed models for overland flow routing of runoff; they are not recommended for channel routing unless the hydograph is very slow rising, the channel slope is moderate to steep, and hydrograph attenuation is quite small. The range of application (with expected modeling errors less than five percent) for kinematic models, including the Muskingum method, is given by the following:

$$T_r S_o^{1.6}/(q_o^{0.2}\, n^{1.2}) \geq 0.014 \tag{26}$$

in which T_r is the time (hrs) of rise of the wave (hydrograph) i.e., the interval of time from beginning of significant rise to when the peak occurs, S_o is the bottom slope (ft/ft), q_o is the unit-width discharge (Q/B) in ft^2/sec, and n is the Manning roughness coefficient (Fread, 1985, 1992).

Diffusion Wave Model

Another simplified distributed routing model, known as the diffusion wave (zero-inertia) model, is based on equation (1) along with an approximation of the momentum equation that retains only the last two terms in equation (2), i.e.,

$$\partial h / \partial x + S_f = 0 \tag{27}$$

Finite-difference approximation techniques, both explicit and implicit (Strelkoff and Katopodes, 1977), have been used to obtain simultaneous solutions to equations (1) and (27). The diffusion simplified routing model considers backwater effects; however, its accuracy is also deficient for very fast rising hydographs, such as those resulting from dam failures, hurricane storm surges, or rapid reservoir releases, which propagate through mild to flat sloping waterways with medium to small Manning's n. The range of application (with expected modeling errors less than 5 percent) for the diffusion models, including the Muskingum-Cunge model, is given by the following (Fread, 1992):

$$T_r S_o^{0.7} n^{0.6} / q_o^{0.4} \geq 0.0003 \tag{28}$$

5.8.4 Dynamic Routing Model

When the complete Saint-Venant equations (1) and (2) are used, the routing model is known as a dynamic routing model. With the advent of high-speed computers, Stoker (1953) and Isaacson et al. (1954, 1956) first attempted to use the complete Saint-Venant equations for routing Ohio River floods. Since then, much effort has been expended on the development of dynamic routing models. Many models have been reported in the literature (Fread, 1985, 1992; Liggett and Cunge, 1975).

Dynamic routing models can be categorized as characteristic or direct methods of solving the Saint-Venant equations. In the characteristic methods, the Saint-Venant equations are first transformed into an equivalent set of four ordinary differential equations which are then approximated with finite differences to obtain solutions. Characteristic methods(Abbott, 1966; Henderson, 1966; Streeter and Wylie, 1967; Baltzer and Lai, 1968) have not proven advantageous over the direct methods for practical flow routing applications.

Direct methods can be classified further as either explicit or implicit. Explicit schemes (Stoker, 1953, 1957; Isaacson et al., 1954; Garrison et al., 1969; Dronkers, 1969; Strelkoff, 1970; Liggett and Cunge, 1975; Veissman et al., 1977; Linsley et al., 1986) transform the differential equations into a set of algebraic equations which are solved sequentially for the unknown flow properties at each cross section at a given time. However, implicit schemes (Preissmann, 1961; Amein and Fang, 1970; Strelkoff, 1970; Fread, 1973, 1977, 1978, 1985; Liggett and Cunge, 1975; Cunge et al., 1980; Shaffranek, 1987; Fread and Lewis, 1988; Chow et al., 1988; Barkow, 1990) transform the Saint-Venant equations into a set of algebraic equations which must be solved simultaneously for all Δx computational reaches at a given time; this set of simultaneous equations may be either linear or nonlinear, the latter requiring an iterative solution procedure.

Explicit methods, although simpler in application, are restricted by numerical stability considerations. Stability problems arise when inevitable errors in computational round-off and those introduced in approximating the partial differential equations via finite differences accumulate to the point that they destroy the usefulness and integrity of the solution, if not the total breakdown of the computations, by creating artificial oscillations of length about $2\Delta x$ in the solution. Due to stability requirements, explicit methods require very small computational time steps on the order of a few seconds or minutes depending on the ratio of the computational reach length (Δx) to the minimum dynamic wave celerity (u), i.e., $\Delta t \le \Delta x/u$, where $u = V + (gA/B)^{1/2}$. This is known as the Courant condition, and it restricts the time step to less than that required for an infinitesimal disturbance to travel the Δx distance. Such small time steps cause explicit methods to be inefficient in the use of computer time.

Implicit finite-difference techniques, however, have no restrictions on the size of the time step due to mathematical stability; however, numerical convergence (accuracy) considerations require some limitation in time step size (Fread, 1974; Cunge et al., 1980). Implicit techniques are generally preferred over explicit because of their computational efficiency. Therefore, an implicit scheme will be subsequently described in detail, herein. Rather than using finite-difference approximation techniques to solve the Saint-Venant equations, finite-element techniques (Gray et al., 1977; DeLong, 1986, 1989) can be used; however, their greater complexity offsets any apparent advantages when compared to a weighted, four-point implicit finite-difference scheme (described later) for solving the one-dimensional flow equations. However, finite-element techniques are often applied to two- and three-dimensional flow computations.

Saint-Venant Equations

A modified and expanded form (Fread, 1988, 1992) of the original one-dimensional Saint-Venant equations (1) and (2) consist of the conservation of mass equation, i.e.,

$$\partial Q/\partial x + \partial s_c(A + A_o)/\partial t - q = 0 \tag{29}$$

and the momentum equation, i.e.,

$$\sigma[\partial(s_m Q)/\partial t + \partial(\beta Q^2/A)/\partial x] + gA(\partial h/\partial x + S_f + S_{ec} + S_i) + L + W_f B = 0 \tag{30}$$

where Q is discharge, h is the water-surface elevation, A is the active cross-sectional area of flow, A_o is the inactive (off-channel storage) cross-sectional area, s_c and s_m are area-weighted and conveyance-weighted sinuosity factors, respectively (DeLong, 1986, 1989) which correct for the departure of a sinuous in-bank channel from the x-axis of the floodplain, x is the longitudinal mean flow-path distance measured along the center of the waterway (channel and floodplain), t is time, q is the lateral inflow or outflow per lineal distance along the waterway (inflow is positive and outflow is negative), σ is a numerical filter ($0 \leq \sigma \leq 1$, usually $\sigma = 1$) to enable the equations to properly handle mixed subcritical/supercritical flows (Fread et al., 1996) during the numerical solution (see a later section on subcritical/supercritical mixed flow for more on σ), β is the momentum coefficient for nonuniform velocity distribution within the cross section, g is the gravity acceleration constant, S_f is the boundary friction slope, S_{ec} is the expansion/contraction (large eddy loss) slope, and S_i is the viscous dissipation slope for mud/debris flows.

Friction Slope. The boundary friction slope (S_f) is evaluated by rearranging the Manning equation (3) for uniform, steady flow into the following form:

$$S_f = n^2|Q|Q/(\mu^2 A^2 R^{4/3}) = |Q|Q/K^2 \tag{31}$$

in which n is the Manning coefficient of frictional resistance (Chow, 1959; Barnes, 1967; Arcement and Schneider, 1984; Jarrett, 1982; and Fread, 1989), R is the hydraulic radius, μ is a units conversion factor (1.49 for US units and 1.0 for SI), and K is the channel conveyance factor. The absolute value of Q is used to correctly account for the possible occurrence of reverse (negative) flows. The conveyance formulation is preferred (for numerical and accuracy considerations) for composite channels having wide, flat overbanks or floodplains in which K represents the sum of the conveyance of the channel (which is corrected for sinuosity effects by dividing by s_m), and the conveyances of left and right floodplain areas.

When the conveyance factor (K) is used to evaluate S_f, the river channel/valley cross-sectional properties are designated as left floodplain, channel, and right floodplain rather than as a composite channel/valley section. Special orientation for designating left or right is not required as long as consistency is maintained. The conveyance factor is evaluated as follows:

$$K = K_\ell + K_c + K_r \tag{32}$$

where:

$$K_\ell = \frac{\mu}{n_\ell} A_\ell R_\ell^{2/3} \tag{33}$$

$$K_c = \frac{\mu\, A_c R_c^{2/3}}{n_c\, s_m^{1/2}} \tag{34}$$

$$K_r = \frac{\mu}{n_r} A_r R_r^{2/3} \tag{35}$$

in which the subscripts ℓ, c, and r designate left floodplain, channel, and right floodplain, respectively.

Sinuosity Factors. The area-weighted and conveyance-weighted sinuosity factors (s_c and s_m, respectively) in equations (29), (30), and (34) represent the ratio(s) of the flow-path distance along a meandering channel to the mean flow-path distance along the floodplain. Initially, only one sinuosity factor (s_k) is specified as varying only with each J[th] depth of flow (J = 1,2,...,$\hat{J}$, where $\hat{J}$ is the number of user-specified tabular top widths (B) vs. h values which describe the cross section geometry), but then this is recomputed within the model according to the following relations:

$$s_{c_J} = \sum_{k=2}^{k=J} \left(\Delta A_{\ell_k} + \Delta A_{c_k} s_k + \Delta A_{r_k}\right) / \left(A_{\ell_J} + A_{c_J} + A_{r_J}\right) \tag{36}$$

$$s_{m_J} = \sum_{k=2}^{k=J} \left(\Delta K_{\ell_k} + \Delta K_{c_k} s_k + \Delta K_{r_k}\right) / \left(K_{\ell_J} + K_{c_J} + K_{r_J}\right) \tag{37}$$

in which $\Delta A = A_{J+1} - A_J$, and s_k represents the sinuosity factor for a differential portion of the flow between the J[th] depth and the J+1[th] depth, and K is the conveyance factor.

Expansion/Contraction Effects. The term (S_{ec}) is computed as follows:

$$S_{ec} = k_{ec} \Delta (Q/A)^2 / (2g\, \Delta x) \tag{38}$$

in which k_{ec} is the expansion/contraction coefficient (negative for expansion/positive for contraction) which varies from -1.0/0.4 for an abrupt change in section geometry to -0.3/0.1 for a very gradual, curvilinear transition between cross sections. The Δ represents the difference in the term $(Q/A)^2$ at two adjacent cross sections separated by a distance Δx. If the flow direction changes from downstream to upstream, k_{ec} can be automatically changed (Fread, 1988).

Large floods such as dam-break-generated floods usually have much greater velocities; it is important, especially for nonuniform channels (Rajar, 1978) to include in the

Saint-Venant momentum equation (30) the expansion/contraction losses via the S_{ec} term defined by equation (38). The ratio of expansion/contraction losses (form losses) to the friction losses can be in the range of $0.01 < S_{ec}/S_f < 1.0$. The larger ratios occur for very irregular channels with relatively small n values and for flows with large velocities (dam-break floods).

Momentum Correction Coefficient. The momentum correction coefficient (β) for nonuniform velocity distribution across the cross section is (Chow, 1959):

$$\beta = \left(K_\ell^2/A_\ell + K_c^2/A_c + K_r^2/A_r\right) / \left[\left(K_\ell + K_c + K_r\right)^2/\left(A_\ell + A_c + A_r\right)\right] \tag{39}$$

in which K is conveyance, A is wetted area, and the subscripts ℓ, c, and r denote left floodplain, channel, and right floodplain, respectively. When floodplain properties are not separately specified and the total cross section is treated as a composite section, β can be approximated as $1.0 \le \beta \le 1.06$ in lieu of equation (39).

Lateral Flow Momentum. The term (L) in equation (30) is the momentum effect of lateral flows, and has the following form (Strelkoff, 1969): (a) lateral inflow, $L = -qv_x$, where v_x is the velocity of lateral inflow in the x-direction of the main channel flow; (b) seepage lateral outflow, $L = -0.5qQ/A$; and (c) bulk lateral outflow, $L = -qQ/A$.

Mud or Debris Flows. The friction loss term (S_i) is included (Fread, 1988) in the momentum equation (30) in addition to S_f to account for viscous dissipation effects of non-Newtonian flows such as mud or debris flows. Also, mine tailings dams, where the viscous contents retained by the dam have non-Newtonian properties, are dam-breach flood applications requiring the use of S_i in equation (30). This effect becomes significant only when the solids concentration of the flow is greater than about 40 percent by volume. For concentrations of solids greater than about 50 percent, the flow behaves more as a landslide and is not governed by the Saint-Venant equations. S_i is evaluated for any non-Newtonian flow as follows (Jin and Fread, 1997):

$$S_i = \frac{\tau_y}{\gamma D}\left[1 + \left(\frac{(b+1)(b+2)\,Q}{(0.74+0.66b)\,(\tau_y/\kappa)^b\,DA}\right)^{\frac{1}{b+0.15}}\right] \tag{40}$$

in which γ is the fluid's unit weight, τ_o is the fluid's yield strength, D is the hydraulic depth (A/B), $b = 1/m$ where m is the exponent of the power function that fits the fluid's stress(τ_s)-strain(dv/dy) properties, and κ is the apparent viscosity or scale factor of the power function, i.e., $\tau_s = \tau_o + \kappa(dv/dy)^m$. The viscous properties, τ_o and κ, can be estimated from the solids concentration ratio of the mud flow (O'Brien and Julien, 1984).

Wind Effects. The last term ($W_f B$) in equation (30) represents the resistance effect of wind on the water surface (Fread, 1985, 1992); B is the wetted topwidth of the active flow portion of the cross section; and $W_f = V_r|V_r|c_w$, where the wind velocity relative to the water is $V_r = V_w \cos w + V$, V_w is the velocity of the wind (+) if opposing the flow velocity and (-) if aiding the flow, w is the acute angle the wind direction makes with the x-axis, V is the velocity of the unsteady flow, and c_w is a wind friction coefficient $(1\,x\,10^{-6} \le c_w \le 3\,x\,10^{-6})$. This modeling capability can be used to simulate the effect of potential dam overtopping due to wind set-up within a reservoir by applying the Saint-Venant equations to the unsteady flow in a reservoir.

Implicit Four-Point, Finite-Difference Approximations

The extended Saint-Venant equations (29) and (30) constitute a system of partial differential equations with two independent variables, x and t, and two dependent variables, h and Q; the remaining terms are either functions of x, t, h, and/or Q, or they are constants. The partial differential equations can be solved numerically by approximating each with a finite-difference algebraic equation; then the system of algebraic equations are solved in conformance with prescribed initial and boundary conditions.

Of various implicit, finite-difference solution schemes that have been developed, a four-point scheme first suggested by Issacson et al. (1954, 1956) and first used by Preissmann (1961) and later by Amein and Fang (1970) and then a weighted version by others (Fread, 1974, 1977, 1985, 1988; Cunge et al., 1980) is most advantageous. It is readily used with unequal distance steps, its stability-convergence properties are conveniently modified, and boundary conditions are easily applied.

Space-Time Plane. In the weighted four-point implicit scheme, the continuous x-t region in which solutions of h and Q are sought is represented by a rectangular grid of discrete points as shown in Fig. 1. An x-t plane (solution domain) is a convenient means for expressing relationships among the variables. The grid points are determined by the intersection of lines drawn parallel to the x- and t-axes. Those parallel to the t-axis represent locations of cross sections; they have a spacing of Δx, which need not be the same between each pair of cross sections. Those parallel to the x-axis represent time lines; they have a spacing of Δt, which also need not be the same between successive time lines. Each point in the rectangular network can be identified by a subscript (i) which designates the x-position or cross section and a superscript (j) which designates the particular time line.

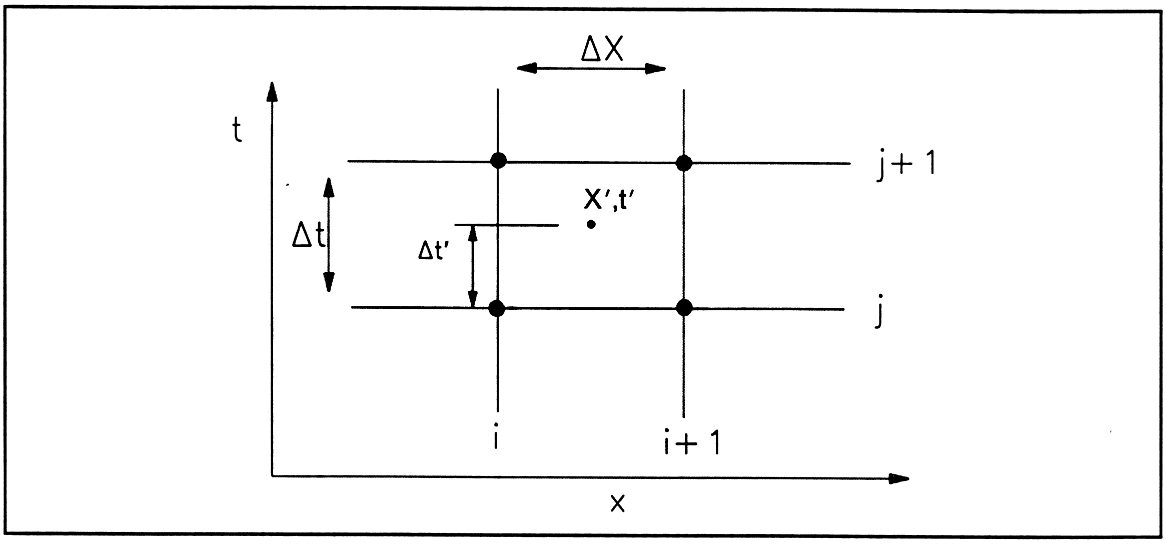

Figure 1. The x-t solution domain for the weighted four-point implicit scheme

Numerical Approximations. The time derivatives are approximated by a forward-difference quotient at point (x',t') centered between the i and i+1 lines along the x-axis as shown in Fig. 1, i.e.,

$$\partial\phi/\partial t \simeq (\phi_i^{j+1} + \phi_{i+1}^{j+1} - \phi_i^j - \phi_{i+1}^j)/(2\ \Delta t_j) \tag{41}$$

where ϕ represents any dependent variable or functional quantity (Q, s_c, s_m, A, A_o, q, h). Spatial derivatives are approximated at point (x',t') by a forward-difference quotient located between two adjacent time lines according to weighting factors of θ (the ratio $\Delta t'/\Delta t$ shown in Fig. 1) and $1-\theta$, i.e.,

$$\partial\phi/\partial x \simeq \theta(\phi_{i+1}^{j+1} - \phi_i^{j+1})/\Delta x_i + (1-\theta)(\phi_{i+1}^j - \phi_i^j)/\Delta x_i \tag{42}$$

Nonderivative terms are approximated with weighting factors at the same time level (point (x',t')) where the spatial derivatives are evaluated, i.e.,

$$\phi \simeq \theta(\phi_i^{j+1} + \phi_{i+1}^{j+1})/2 + (1-\theta)(\phi_i^j + \phi_{i+1}^j)/2 \tag{43}$$

The weighted four-point implicit scheme is unconditionally, linearly stable for $\theta \geq 0.5$ (Fread, 1974); however, the sizes of the Δt and Δx computational steps are limited by the accuracy of the assumed linear variations of functions between the grid points in the x-t solution domain. Values of θ greater than 0.5 dampen parasitic oscillations which have wave lengths of about $2\Delta x$ that can grow enough to invalidate or destroy the solution. The θ weighting factor causes some loss of accuracy as it departs from 0.5, a box scheme, and approaches 1.0, a fully implicit scheme. This effect becomes more pronounced as the magnitude of the ratio ($T_r/\Delta t$) decreases where T_r is the time of rise of the hydrograph (time interval from beginning of significant rise to peak of the hydrograph). Usually, a θ weighting factor of 0.60 is used to minimize the loss of accuracy while avoiding the possibility of weak (pseudo) instability for θ values of 0.5 when frictional effects are minimal.

Selection of Δt and Δx Computational Parameters. The computational time step (Δt) can be either specified or automatically determined to best suit the most rapidly rising hydrograph occurring within a system of rivers that may contain one or more breaching dams or other dynamic internal boundary conditions. The time step is selected according to the following:

$$\Delta t = T_r / M \tag{44}$$

where T_r is the minimum time of rise (seconds) of any hydrograph that has been specified at upstream boundaries or in the process of being generated at a breaching dam. M is user specified according to the following guidance (Fread, 1993):

$$M \simeq 2.67 \left[1 + \mu' n^{0.9} / (q^{0.1} S_o^{0.45})\right] \tag{45}$$

in which $\mu' = 3.97$ (3.13 SI units), n is the Manning friction coefficient, q is the peak flow per unit channel width (Q/B), and S_o is the channel bottom slope. M usually varies within the range, $6 \le M \le 40$, with M often assumed to be approximately 20.

The Δx computational distance step can be specified or automatically determined according to the smaller of two criteria (Fread, 1993). The first criterion is:

$$\Delta x \le c T_r / 20 \tag{46}$$

in which c is the bulk wave celerity (the celerity or velocity associated with an essential characteristic of the unsteady flow such as the peak of the hydrograph). In most applications, the wave velocity is well approximated as a kinematic wave, and c is estimated as $3/2V$ (V is the flow velocity) or c can be obtained by dividing the distance between two points along the channel by the difference in the times of occurrence of the peak of an observed or computed flow hydrograph at each point. Since c can vary along the channel, and depending upon the extent of this variation, Δx may not be constant along the channel.

The second criterion for selecting Δx is the restriction imposed by rapidly varying cross-sectional changes along the x-axis of the waterway. Such expansion/contraction is limited to the following inequality (Samuels, 1985):

$$0.635 < A_{i+1}/A_i < 1.576 \tag{47}$$

This condition results in the following approximation (Fread, 1988) for the maximum computational distance step:

$$\Delta x \le L'/N' \tag{48}$$

where:

$$N' = 1 + 2\ |A_i - A_{i+1}|/\hat{A} \tag{49}$$

in which L' is the distance between two adjacent (i and $i+1$) cross sections differing from one another by approximately 50 percent or greater, A is the active cross-sectional area, $\hat{A} = A_{i+1}$ if $A_i > A_{i+1}$ (contracting reach) or $\hat{A} = A_i$ if $A_i < A_{i+1}$ (expanding reach), and N' is rounded to the nearest integer value.

Significant changes in the bottom slope of the waterway also require small distance steps in the vicinity of the change. This is required particularly when the flow changes from subcritical to supercritical or conversely along the waterway. Such changes can require computational distance steps in the range of 50 to 200 ft (15 to 63 m).

Automatic Interpolation. It is convenient to automatically provide linearly interpolated cross sections at a user specified spatial resolution in order to increase the spatial frequency at which solutions to the Saint-Venant equations are obtained. This is often required for purposes of attaining numerical accuracy/stability when (a) routing very sharp-peaked hydrographs such as those generated by breached dams, (b) when adjacent cross sections either expand or contract by more than about 50 percent, and (c) where mixed flow changes from subcritical to supercritical or vice versa.

Algebraic Routing Equations. Using the finite-difference operators of equations (41)-(43) to replace the derivatives and other variables in equations (29) and (30), the following weighted four-point, implicit, nonlinear, finite-difference algebraic equations are obtained:

$$\theta\left[\frac{Q_{i+1}^{j+1}-Q_i^{j+1}}{\Delta x_i}\right]-\theta\, q_i^{j+1}+(1-\theta)\left[\frac{Q_{i+1}^{j}-Q_i^{j}}{\Delta x_i}\right]-(1-\theta)\, q_i^{j}+$$

$$\left[\frac{s_{c_i}^{j+1}(A+A_o)_i^{j+1}+s_{c_i}^{j+1}(A+A_o)_{i+1}^{j+1}-s_{c_i}^{j}(A+A_o)_i^{j}-s_{c_i}^{j}(A+A_o)_{i+1}^{j}}{2\Delta t_j}\right]=0 \tag{50}$$

$$\sigma\left[\frac{\left(s_{m_i}Q_i\right)^{j+1}+\left(s_{m_i}Q_{i+1}\right)^{j+1}-\left(s_{m_i}Q_i\right)^{j}-\left(s_{m_i}Q_{i+1}\right)^{j}}{2\Delta t_j}\right]+\theta\left[\sigma\left(\frac{(\beta Q^2/A)_{i+1}^{j+1}-(\beta Q^2/A)_i^{j+1}}{\Delta x_i}\right)\right.$$

$$\left.+\, g\,\bar{A}_i^{j+1}\left(\frac{h_{i+1}^{j+1}-h_i^{j+1}}{\Delta x_i}+\bar{S}_{f_i}^{j+1}+S_{ec_i}^{\;j+1}+S_{i_i}^{j+1}\right)+L_i^{j+1}+(W_f\bar{B})_i^{j+1}\right]+(1-\theta) \tag{51}$$

$$\left[\sigma\left(\frac{(\beta Q^2/A)_{i+1}^{j}-(\beta Q^2/A)_i^{j}}{\Delta x_i}\right)+g\,\bar{A}_i^{j}\left(\frac{h_{i+1}^{j}-h_i^{j}}{\Delta x_i}+\bar{S}_{f_i}^{j}+S_{ec_i}^{\;j}+S_{i_i}^{j}\right)+L_i^{j}+(W_f\bar{B})_i^{j}\right]=0$$

where:

$$\bar{A}_i=(A_i+A_{i+1})/2 \tag{52}$$

$$\bar{S}_{f_i}=n^2\,\bar{Q}_i\left|\bar{Q}_i\right|/(\mu^2\,\bar{A}_i^2\,\bar{R}_i^{4/3})=\bar{Q}_i\left|\bar{Q}_i\right|/\bar{K}_i^2 \tag{53}$$

$$\bar{Q}_i=(Q_i+Q_{i+1})/2 \tag{54}$$

$$\bar{R}_i\simeq\bar{A}_i/\bar{B}_i \tag{55}$$

$$\bar{B}_i=(B_i+B_{i+1})/2 \tag{56}$$

$$\bar{K}_i=(K_i+K_{i+1})/2 \tag{57}$$

The terms L and W_fB are defined in equation (30); terms associated with the j^{th} time line are known from initial conditions or previous time-step computations; and μ in equation (53) is defined in equation (31). The Δx distance between cross sections is measured along the peak flow path through the waterway.

Solution Procedure

The flow equations are expressed in finite-difference form for all Δx_i reaches between the first and last (N-th) cross section ($i = 1,2,\ldots,N$) along the channel/floodplain and then solved simultaneously for the unknowns (Q and h) at each cross section. In essence, the solution technique determines the unknown quantities (Q and h at all specified cross sections along the waterway) at various times into the future; the solution is advanced from one time to a future time over a finite time interval (time step) of magnitude Δt. Thus, applying equations (50) and (51) recursively to each of the (N-1) rectangular grids in Fig. 1 between the upstream and downstream boundaries, a total of (2N-2) equations with 2N unknowns are formulated. Then, prescribed boundary conditions for subcritical flow (Froude number less than unity, i.e., $Fr = Q/(A\sqrt{gD}) < 1$), one at the upstream boundary and one at the downstream boundary, provide the two additional and necessary equations required for the system to be determinate. Since disturbances can propagate only in the downstream direction in supercritical flow ($Fr > 1$), two upstream boundary conditions and no downstream boundary condition are required for the system to be determinate when the flow is supercritical throughout the routing reach. The boundary conditions are described later. Due to the nonlinearity of equations (50) and (51) with respect to Q and h, an iterative, highly efficient quadratic solution technique such as the Newton-Raphson method is frequently used. Other solution techniques linearize equations (50) and (51) via a Taylor series expansion or other means. Convergence of the iterative technique is attained when the difference between successive solutions for each unknown is less than a relatively small prescribed tolerance. Convergence for each unknown at all cross sections is usually attained within about one to five iterations with the majority of solutions obtained within two iterations. A more complete description of the solution method may be found elsewhere (Fread, 1985).

The solution of 2N x 2N simultaneous equations requires an efficient matrix technique for the implicit method to be feasible. One such procedure requiring 38N computational operations (+, -, *, /) is a compact, penta-diagonal Gaussian elimination method (Fread, 1971, 1985) which makes use of the banded structure of the coefficient matrix of the system of equations. This is essentially the same as the double sweep elimination method (Liggett and Cunge, 1975; Cunge et al., 1980).

When flow is everywhere and at all times supercritical, the solution technique previously described can be somewhat simplified. Two boundary conditions are required at the upstream boundary and none at the downstream boundary since flow disturbances cannot propagate upstream in supercritical flow. The unknown h and Q at the most upstream cross section are determined from the two boundary equations. Then, cascading from upstream to downstream, equations (50) and (51) are solved for the two unknowns (h_{i+1} and Q_{i+1}) at each cross section by using Newton-Raphson iteration applied recursively to the two nonlinear equations, with $\sigma = 1$ in equation (51).

Initial Conditions

Values of water-surface elevation (h) and discharge (Q) for each cross section must be specified initially at time t = 0 to obtain solutions to the Saint-Venant equations. Initial conditions may be obtained from any of the following: (a) observations at gaging stations and using interpolated values between gaging stations for intermediate cross sections in large rivers; (b) computed values from a previous unsteady flow solution (used in real-time flood forecasting); and (c) computed values from a steady-flow backwater solution. The backwater method is most commonly used, in which the steady discharge at each cross section is determined by:

$$Q_{i+1} = Q_i + q_i \, \Delta x_i \qquad \ldots \; i = 1,2,3,\ldots,N-1 \tag{58}$$

in which Q_1 is the assumed steady flow at the upstream boundary at time $t=0$, and q_i is the known average lateral inflow or outflow along each Δx reach at $t=0$. The water-surface elevations (h_i) are computed according to the following steady-flow simplification of the momentum equation (30):

$$(Q^2/A)_{i+1} - (Q^2/A)_i + g\bar{A}_i \, (h_{i+1} - h_i + \Delta x_i \, \bar{S}_{f_i}) = 0 \tag{59}$$

in which $\bar{A}$ and $\bar{S}_f$ are defined by equations (52) and (53), respectively. The computations proceed in the upstream direction (i = N-1, ... 3,2,1) for subcritical flow (they must proceed in the downstream direction for supercritical flow). The starting water-surface elevation (h_N) can be specified or obtained from the appropriate downstream boundary condition for the discharge (Q_N) obtained via equation (58). The Newton-Raphson iterative solution method (Fread and Harbaugh, 1971) for a single equation and/or a simple, less efficient, but more stable bi-section iterative technique can be applied to equation (59) to obtain h_i. Due to friction, small errors in the initial conditions will dampen-out after several computational time steps during the solution of the Saint-Venant equations.

Upstream Boundary

Values for the unknowns at external boundaries (the upstream and downstream extremities of the routing reach) of the channel/floodplain, must be specified in order to obtain solutions to the Saint-Venant equations. In fact, in most unsteady flow applications, the unsteady disturbance is introduced at one or both of the external boundaries.

A specified discharge time series (hydrograph) of inflow to the most upstream cross section is used as the upstream boundary condition. The hydrograph should not be affected by downstream flow conditions. This hydrograph may be obtained from the following: (1) historical observations, (2) assumed design hydrograph, or (3) a runoff hydrograph from specified rainfall-runoff model using calibrated or estimated model parameters. The upstream boundary is expressed mathematically as follows:

$$Q_1^{j+1} - Q(t) = 0 \tag{60}$$

in which $Q(t)$ is the specified discharge time series and the subscript indicates the discharge at the first cross section, i.e., the upstream boundary. Equation (60) is used for the upstream boundary if dynamic routing (based on the discretized Saint-Venant equations) commences at this location. However, if the most upstream cross section represents the inlet to an upstream reservoir, a simple routing procedure (reservoir level-pool routing) can be used rather than the considerably more complex dynamic routing if (1) the reservoir is not excessively long and (2) the inflow hydrograph $Q(t)$ is not rapidly changing with time. Level-pool routing errors (Fread, 1992), with a magnitude of less than about 5 percent, can usually be tolerated.

Downstream Boundary

For subcritical flow, a specified discharge or water-surface elevation time series, or a tabular relation between discharge and water-surface elevation (single-valued rating curve) can be used as the downstream boundary condition.

Another downstream boundary condition can be a computed loop-rating curve based on the Manning equation, i.e.,

$$Q_N^{j+1} - \mu/n \, A_N^{j+1} \left(R_N^{j+1}\right)^{2/3} \left(S_{f_N}^{j}\right)^{1/2} = 0 \tag{61}$$

The loop is produced by using the friction slope (S_f) rather than the channel bottom slope (S_o) in the Manning equation. The friction slope exceeds the bottom slope during the rising limb of the hydrograph while the reverse is true for the recession limb. The friction slope (S_f) is approximated by using equation (30) where L and W_f are assumed to be zero while s_m and β are assumed to be unity (Fread, 1985, 1988, 1992), i.e.,

$$S_{f_N}^j \simeq -\left(Q_N^j - Q_N^{j-1}\right)/\left(gA_N^j\,\Delta t^j\right) - \left[\left(Q^2/A\right)_N^j - \left(Q^2/A\right)_{N-1}^j\right]/\left(gA_N^j\,\Delta x_{N-1}\right) - \left(h_N^j - h_{N-1}^j\right)/\,\Delta x_{N-1} \tag{62}$$

The loop-rating boundary equation allows the unsteady wave to pass the downstream boundary with minimal disturbance by the boundary itself, which is desirable when the routing is terminated at an arbitrary location along the channel/floodplain and not at a location of actual flow control such as a dam or waterfall, or where the flow is affected by downstream backwater conditions produced by tidal action, reservoirs, or tributary inflow.

When the downstream boundary is a stage/discharge relation (rating curve), the flow at the boundary should not be otherwise affected by flow conditions further downstream. Although there are often some minor effects due to the presence of cross-sectional irregularities downstream of the chosen boundary location, these usually can be neglected unless the irregularity is so pronounced as to cause significant backwater or drawdown effects. Reservoirs, major tributaries, or tidal effects located below the downstream boundary which cause backwater effects at the boundary should be avoided. When either of these situations are unavoidable, the routing reach should be extended downstream to the dam in the case of the reservoir or to a location downstream of where the major tributary enters. Sometimes the routing reach may be shortened by moving the downstream boundary to a location further upstream where backwater effects are negligible.

Internal Boundaries

Often along the channel/floodplain, there are locations such as a dam, bridge, or waterfall (short rapids) where the flow is rapidly varied in space rather than gradually varied. At such locations (internal boundaries), the Saint-Venant equations are not applicable since gradually varied flow is a necessary condition for their derivation. Empirical water elevation-discharge relations such as weir-flow are utilized for simulating rapidly varying flow. At internal boundaries, cross sections are specified for the upstream and downstream extremities of the section where rapidly varying flow occurs. The Δx reach containing an internal boundary requires two internal boundary equations; since, as with any other Δx reach, two equations equivalent to the Saint-Venant equations are required. One of the required internal boundary equations represents conservation of mass with negligible time-dependent storage, i.e.,

$$Q_i^{j+1} - Q_{i+1}^{j+1} = 0 \tag{63}$$

Dam. The second equation is usually an empirical rapidly varied flow relation. If the internal boundary represents a dam, the following equation can be used:

$$Q_i^{j+1} - (Q_s + Q_b)^{j+1} = 0 \tag{64}$$

in which Q_s and Q_b are the spillway and dam-breach flow, respectively. In this way, the flows Q_i and Q_{i+1} and the elevations h_i and h_{i+1} are in balance with the other flows and elevations occurring simultaneously throughout the entire flow system which may consist of additional downstream dams which are treated as additional internal boundary conditions via equations (63) and (64). In fact, this approach can be used to simulate the progression of a dam-break flood through an unlimited number of reservoirs located sequentially along the valley. The downstream dams may also breach if they are sufficiently overtopped. The spillway flow (Q_s) is computed from the following expression:

$$Q_s = c_s L_s (h_i - h_s)^{1.5} + c_g A_g (h_i - h_g)^{0.5} + c_d L_d (h_i - h_d)^{1.5} + Q_t \tag{65}$$

in which c_s is the uncontrolled spillway discharge coefficient, h_s is the uncontrolled spillway crest, c_g is the gated spillway discharge coefficient, h_g is the center-line elevation of the gated spillway, c_d is the discharge coefficient for flow over the crest of the dam, L_s is the spillway length, and Q_t is a constant outflow term which is head independent or it may be a specified discharge time series. The uncontrolled spillway flow or the gated spillway flow can also be represented as a table of head-discharge values. The gate flow may also be specified as a function of time via a known time series for $A_g(t)$. The breach outflow (Q_b) is computed as broad-crested weir flow (Fread, 1977, 1985, 1988, 1992; Fread and Lewis, 1988), i.e.,

$$Q_b = c_v k_s [3.1\ b_i\ (h_i - h_b)^{1.5} + 2.45\ z\ (h_i - h_b)^{2.5}] \tag{66}$$

in which c_v is a small correction for velocity of approach, b_i is the instantaneous breach bottom width, h_i is the elevation of the water surface just upstream of the structure, h_b is the elevation of the breach bottom in which h_b is assumed to be a linear or nonlinear function of time (t_b) from beginning of the breach formation time (τ), z is the side slope of the breach, and k_s is the submergence correction factor due to the downstream tailwater elevation (h_t), i.e.,

$$k_s = 1.0 \qquad h^* \le 0.67 \tag{67}$$

$$k_s = 1.0 - 22.3(h^* - 0.67)^3 \qquad h^* > 0.67 \tag{68}$$

where:

$$h^* = (h_t - h_b)/(h_i - h_b) \tag{69}$$

Using a parametric description of the breach, the instantaneous breach bottom width (b_i) starts at a point at the crest of the dam and enlarges at a linear or nonlinear rate over the failure time (τ) until the terminal bottom width (b) is attained and the breach bottom has eroded to the minimum elevation, h_{bm}. The instantaneous bottom elevation of the breach (h_b) is described as a function of time (t_b) according to the following:

$$h_b = h_d - (h_d - h_{bm})\,(t_b/\tau)^\rho \qquad 0 \le t_b \le \tau \tag{70}$$

in which h_d is the elevation of the top of the dam, h_{bm} is the final elevation of the breach bottom which is usually, but not necessarily, the bottom of the reservoir or outlet channel bottom, t_b is the time since beginning of breach formation, and ρ is the parameter specifying the degree of nonlinearity, e.g., $\rho=1$ is a linear formation rate, while $\rho=2$ is a nonlinear quadratic rate; the range for ρ is $1 \le \rho \le 4$, with the linear rate usually assumed. The interval of time (τ) required for the breach to form is given by $\tau = 0.3\,V_r^{0.53}/H_d^{0.9}$ in which $H_d = h_b - h_{bm}$, V_r is the reservoir volume (acre-ft) from empirical data by Froehlich (1987); the standard error of estimate for τ is $\pm$ 0.9 hrs or $\pm$ 74 percent of τ (Fread, 1988, 1995). The instantaneous bottom width (b_i) of the breach is given by the following:

$$b_i = b(t_b/\tau)^\rho \qquad 0 \le t_b \le \tau \tag{71}$$

in which b is the final width of the breach bottom given by $b = \bar{b} - zH_d$ and $\bar{b} = 9.5\,k_o\,(V_r H_d)^{0.25}$ from empirical data by Froehlich (1987) in which $k_o = 0.7$ for piping and $k_o = 1.0$ for overtopping; the standard error of estimate for b is ± 82 ft or $\pm$ 56 percent of b (Fread,1988, 1995).

When simulating a dam failure, the actual breach formation can commence when the reservoir water-surface elevation (h) exceeds a user-specified value, h_f. This feature permits the simulation of an overtopping of a dam in which the breach does not form until a sufficient amount of water has passed over the crest of the dam to have eroded away the downstream face of the dam.

If the breach is formed by piping, equation (66) is replaced by an orifice equation:

$$Q_b = 4.8\,A_p(h_i - h_p)^{1/2} \tag{72}$$

where:

$$A_p = [b_i + z(h_p - h_b)](h_p - h_b) \tag{73}$$

in which h_p is the specified center-line elevation of the pipe. Each of the terms in equation (65) except Q_t may be modified by a submergence correction factor similar to k_s which can be computed by equations (67) - (69), but in equation (69) h_b is replaced by h_s, h_g, and h_d, respectively.

Bridge. If the internal boundary represents highway/railway bridges together with their earthen embankments which cross the floodplain, equations (63) and (64) can still be used although Q_s in equation (65) is computed by the following contracted bridge flow expression:

$$Q_s = C_b\sqrt{g}\, A_{i+1}(h_i - h_{i+1})^{0.5} + C_d k_s (h_i - h_c)^{1.5} \tag{74}$$

in which C_b is a coefficient of bridge flow (Chow, 1959), C_d is the coefficient of flow over the crest of the road embankment, h_c is the crest elevation of the embankment, and k_s is similar to equations (67)-(69) except h_b is replaced by h_c. A breach of the embankment is treated the same as with dams.

Levee Overtopping/Floodplain Interactions

Flows which overtop levees located along either or both sides of a main-stem river and/or its tributaries can be treated as lateral flow (q) in equations (29) and (30) where the lateral flow diverted over the levee is computed as broad-crested weir flow. This overtopping flow is corrected for submergence effects if the floodplain water-surface elevation sufficiently exceeds the levee crest elevation. After the flood peak passes, the overtopping flow may reverse its direction when the floodplain water-surface elevation exceeds the river water-surface elevation, thus allowing flow to return to the river. The overtopping broad-crested weir flow is computed according to the following:

$$q = -c_\ell\, k_s\, (h - h_c)^{3/2} \tag{75}$$

where k_s, the submergence correction factor, is computed as in equations (67)-(69) except $h^* = (h_{fp} - h_c)/(h - h_c)$, in which c_ℓ is the weir discharge coefficient, h_c is the levee-crest elevation, h is the water-surface elevation of the river, and h_{fp} is the water-surface elevation of the floodplain. Flow in the floodplain can affect overtopping flows via the submergence correction factor. Flow may also pass from the waterway to the floodplain through a time-dependent crevasse (breach) in the levee via a breach-flow equation similar to equation (66). The floodplain, which is separated from the principal routing channel (river) by the levee, may be treated as: (a) a dead-storage area (A_o) in the Saint-Venant equations, in which case equation (75) is not relevant; (b) a tributary which receives its inflow as lateral flows (the flows from the river which overtop the levee-crest) which are simultaneously dynamically routed along the floodplain; and (c) the flows and water-surface elevations can be computed by using a level-pool routing method particularly if the floodplain is divided into compartments by levees (dikes) or elevated roadways located somewhat perpendicular to the river levee(s).

Supercritical/Subcritical Mixed Flows

Flow can change with either time or distance along the routing reach from supercritical to subcritical while passing through critical flow, or conversely. This "mixed flow" requires special treatment to prevent numerical instabilities in the solution of the Saint-Venant equations. Such a treatment for mixed flows (Fread et al., 1996) is to provide a "local partial inertia" filter, i.e.,

$$\sigma = (1 - Fr^{m}) \tag{76}$$

which multiplies the first two (inertia) terms in the momentum equation (30) and equation (51). Fr is the Froude number of the flow in any i^{th} Δx reach, and the exponent (m) varies from 1 to 10, with 5 usually preferred. The filter takes on a value of zero when $Fr \geq 1$. The local partial inertia filter (σ) avoids numerical difficulties associated with mixed flows while introducing negligible errors, less than about a maximum of 2 percent for all flows and less than 1 percent for almost all flow conditions.

Flow Through a System of Rivers

A river system consisting of a main-stem river and one or more tributaries is efficiently solved using an iterative relaxation method (Fread, 1973, 1985) in which the flow at the confluence of the main-stem and tributary is treated as the lateral inflow/outflow (q) in equations (29) and (30). This algorithm was extended so as to treat a dendritic system of waterways having n^{th}-order tributaries (Lewis et al., 1996). If the river system has any bifurcations such as islands, along with or without n^{th}-order tributaries, a network solution technique is used (Fread, 1985), wherein three internal boundary equations conserve mass and momentum at each bifurcation or junction confluence. The resulting system of algebraic equations uses a special sparse matrix Gaussian elimination technique for an efficient solution (Fread, 1983).

5.8.5 References

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